Option Pricing Model for Autocorrelated Stock Returns

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ABSTRACT

This paper develops an analytic option pricing model for the case of serially correlated asset returns. This model provides a valuable insight into dependence of option price on the return autocorrelation. A surprising relationship is found between asset price volatility, asset return volatility and asset return autocorrelation coefficient. The analytical solution obtained here reduces to the well known Black Scholes option pricing formula for the special case of no autocorrelation in asset returns. However, in the case of serially correlated asset returns, our model provides superior results by controlling for the difference between asset price volatility and asset return volatilities. Empirical tests obtain statistically and economically significant results demonstrating the superiority of our model when compared to the traditional Black Scholes formula. Additionally, a Monte Carlo method is developed to price option for assets with autocorrelated returns. Numerical comparison between these two methods demonstrate strikingly similarity, being equal to within the computational error. This serves to affirm the validity of our method.

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Past research has uncovered significant predictability in short and long-term stock returns. For the short-term periods Lo and KacKinlay (1988) find that weekly returns on portfolios of NYSE stocks show reliable positive autocorrelation. To mitigate the nonsynchronous trading problem, Conrad and Kaul (1988) examine the autocorrelation of Wednesday-Wednesday returns for size grouped portfolios and find first order autocorrelation of weekly returns varying between 0.09 to 0.3. For longer periods Fama and French (1988) find that autocorrelation of returns on diversified portfolios of NYSE stocks becomes strongly negative. Similarly Peterba and Summers (1988) find that the variance on diversified portfolios grows less than in proportion to N (length of observation), this finding is consistent with the hypothesis of negative serial correlation.

The evidence of persistent autocorrelation in asset returns contradicts the assumption made in deriving option pricing models. Current option pricing models, regardless of the framework (i) the stochastic-interest rate option models of Merton (1973) and Amin and Jarrow (1992); (ii) the jump-diffusion/pure jump models of Bates(1991), Madan and Chang (1996), and Merton (1976); (iii) the constant-elasticity-of-variance model of Cox and Ross (1976); (iv) the stochastic volatility models of Heston (1993), Hull and White (1987), Melino and Turnbull (1990, 1995), Scott(1987); (v) stochastic-volatility and stochastic-interest rates models of Amin and Ng(1993), Bakshi and Chen (1997); (vi) stochastic volatility jump-diffusion models of Bates (1996a), (this list is by no means exhaustive) assume that asset returns are distributed independently of each other.

In this paper we are going to create a framework of a lognormally distributed asset price S with serially correlated returns and derive an analytic option pricing model, capable of providing an exact solution for a value of a derivatives on such an asset. We will develop a framework of random, normally distributed, process x, such that $lnS = x$, with autocorrelated increments ζ that have volatility σ^2 and autocorrelation coefficient ρ. Both parameters can be estimated using historical data, Hull (1999), Andrews (1993). Autocorrelated returns ζ will be constructed using independent identically distributed (iid) normal random variables ε with arbitrary volatility σ_ε^2.

The results obtained from our model show that presence of return autocorrelation affects the volatility and expected value of asset price, making them functions of correlation coefficient and time to expiration. We find that within our framework the asset price volatility can no longer be expressed as $\sigma^2 t$ where σ^2 is the variance of asset price returns. This finding implies that the traditional approach to estimating asset price volatility contains bias proportional to autocorrelation coefficient.

We create a Monte Carlo procedure capable of pricing options in the presence of autocorrelation in asset returns. The simulation results are compared with the analytical results. A striking accuracy is obtained in this comparison. The values obtained using both methods are identical to within the rounding error (0.1%).

The rest of this article of organized as follows: Section I develops the framework for a process with autocorrelated increments. Section II derives an analytical option pricing model by solving a second order differential equation. Section III provides an alternative derivation of an analytical option pricing formula within the framework of Section I using an integral representation of the problem. Section IV lists some of the properties of this option pricing model and evaluates its performance using stock market data. Section V develops numerical Monte Carlo procedure to model the option price process with autocorrelation in asset returns, additionally the results of this simulation are compared with the analytical results obtained in Sections II and (III) to illustrate the advantages of new analytical model. Concluding remarks and avenues for further research are offered in Section VI. Proofs of equations are provided in Appendix.

I. Serially Correlated Asset Price Process

For the analytical work to follow we would like to construct a framework of normally distributes variables x having autocorrelated increments with volatility σ^2 and autocorrelation coefficient ρ.

Let x be defined by the following stochastic equation.

$$\Delta x_n = \sqrt{\Delta t} \cdot \zeta_n(\sigma^2, \rho). \tag{1}$$

Values of x will follow a random walk with normally distributed autocorrelated increments ζ that have volatility σ^2 and autocorrelation coefficient ρ measured over time period Δt. Increments ζ would be constructed using iid normal variables ε. Therefore a stable AR(1) process of the form

$$\zeta_n(\sigma^2, \rho) = \alpha \cdot \zeta_{n-1}(\sigma^2, \rho) + \varepsilon_n(\sigma_\varepsilon^2), \tag{2}$$

will be used to describe the increments ζ, where $\varepsilon_n(\sigma_\varepsilon^2)$ are normal and iid. Equation (2) has two unknown parameters: α and σ_ε^2. Both of them can be found by enforcing the mean and volatility conditions, i.e. the volatility must equal σ^2 and $\mu_\zeta = 0$. Solving the resulting equations and substituting their values back into original equation (2) gives

$$\zeta_n(\sigma^2, \rho) = \rho \zeta_{n-1}(\sigma^2, \rho) + \varepsilon_n((1-\rho)^2 \sigma^2). \tag{3}$$

Therefore, in order for stochastic increments ζ to have a volatility of σ^2, the stochastic term ε must be normally distributed with volatility σ_ε^2 equal to $(1-\rho^2)\sigma^2$. As is well known, autocorrelation coefficient for AR(1) model is equal to

$$\rho_n = \rho_1^n \equiv \rho^n. \tag{4}$$

A correlation decay time τ_{corr} is introduced as

$$|\rho| \sim exp(-t/\tau_{corr}). \tag{5}$$

Correlation decay time is equal to the time interval required for the correlation coefficient to decrease by the factor of $e \approx 2.73$. Expression (6) can alternatively be written as

$$\tau_{corr} = \frac{\Delta t}{ln(1/|\rho|)} \geq 0. \tag{6}$$

where Δt represents the time increments used to measure the correlation coefficient ρ and volatility σ^2. According to Appendix A, for small values of n, such that $n < 1/ln(1/|\rho|)$, the volatility of ζ_n will differ from σ^2. In order to satisfy the requirement of $Vol(\zeta) = \sigma^2$, this process must starts at point $n = -k$ for some large positive k (usually $k \approx 20-30$). This does not conflict with estimating the value of σ^2 from historical data. The length of time traditionally used to estimate the volatility of ζ will provide sufficiently large number of observations to obtain $Vol(\zeta) = \sigma^2$.

For the purposes of option pricing we are only interested in the weak-form solution of equation (1) that provides the distribution function at a particular time t_n. The weak-form solution of (1) is found to be

$$x_n \sim N(0, \Psi^2(n, \rho)), \tag{7}$$

where

$$\Psi^2(n, \rho) = \sigma^2_{eff}(\rho)t + \sigma^2_{corr}(\rho)\tau_{corr}, \tag{8}$$

and

$$\sigma^2_{eff}(\rho) = \frac{1+\rho}{1-\rho}\sigma^2, \tag{9}$$

$$\sigma^2_{corr}(\rho) = -(1-\rho^n) \cdot sign(\rho) \cdot \sigma^2_{eff}(\rho), \tag{10}$$

where $sign(\rho)$ is a standard sign function. For the continuous time process with positive correlation, equation (10) can be written as:

$$\sigma^2_{corr}(\rho) = -(1-\rho^n)\sigma^2_{eff}(\rho) = -\left[1 - exp(-t/\tau_{corr})\right]\sigma^2_{eff}(\rho). \tag{11}$$

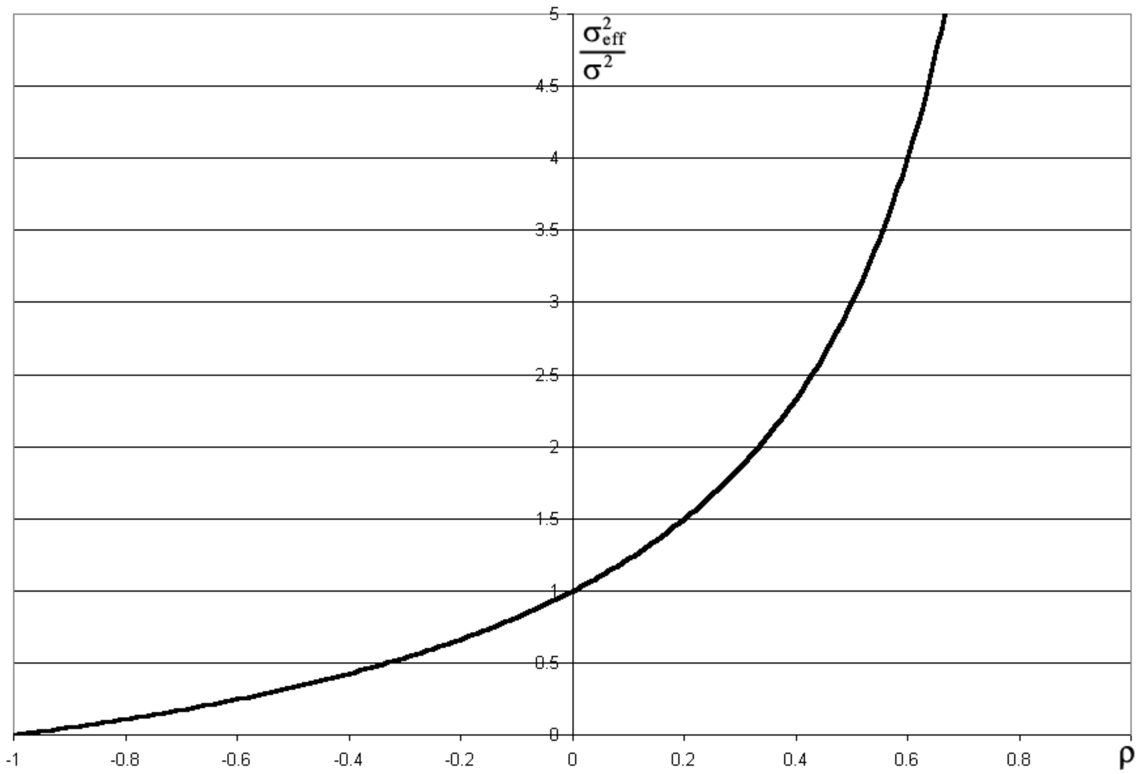

Figure 1. Ratio of σ^2_{eff} vs σ^2 as a function of autocorrelation coefficient ρ.

The two components of (8) deserve special attention. Time derivative of $\Psi^2(n,\rho)$ for $n >> 1$ is equal to $\sigma^2_{eff}(\rho)$. This value can be greater than σ^2 for positive serial correlation or less than it for negative serial correlation as shown in Figure 1. The second term $\sigma^2_{corr}(\rho)\tau_{corr}$ can further be split into two components: one is constant and equal to $\sigma^2_{eff} \cdot sign(\rho) \cdot \tau_{corr}$, the second one is equal to $exp(-t/\tau_{corr}) \cdot \sigma^2_{eff}\tau_{corr}$. This second term decays exponentially with time and can be neglected for $t >> \tau_{corr}$ but it is very significant for the time periods $t \leq \tau_{corr}$.

An analysis of the behavior of $\Psi^2(n,\rho)$ yields another interesting result. For $t \to 0$, $\Psi^2(1,\rho) \to 0$, and it asymptotically converges to $\sigma^2_{eff}(t - sign(\rho)\cdot\tau_{corr})$ for $t > 0$. In other words, the initial value of asset price volatility is equal to 0 and converges to its assymptotic limit during time interval equal to a two-three correlation decay times τ_{corr}

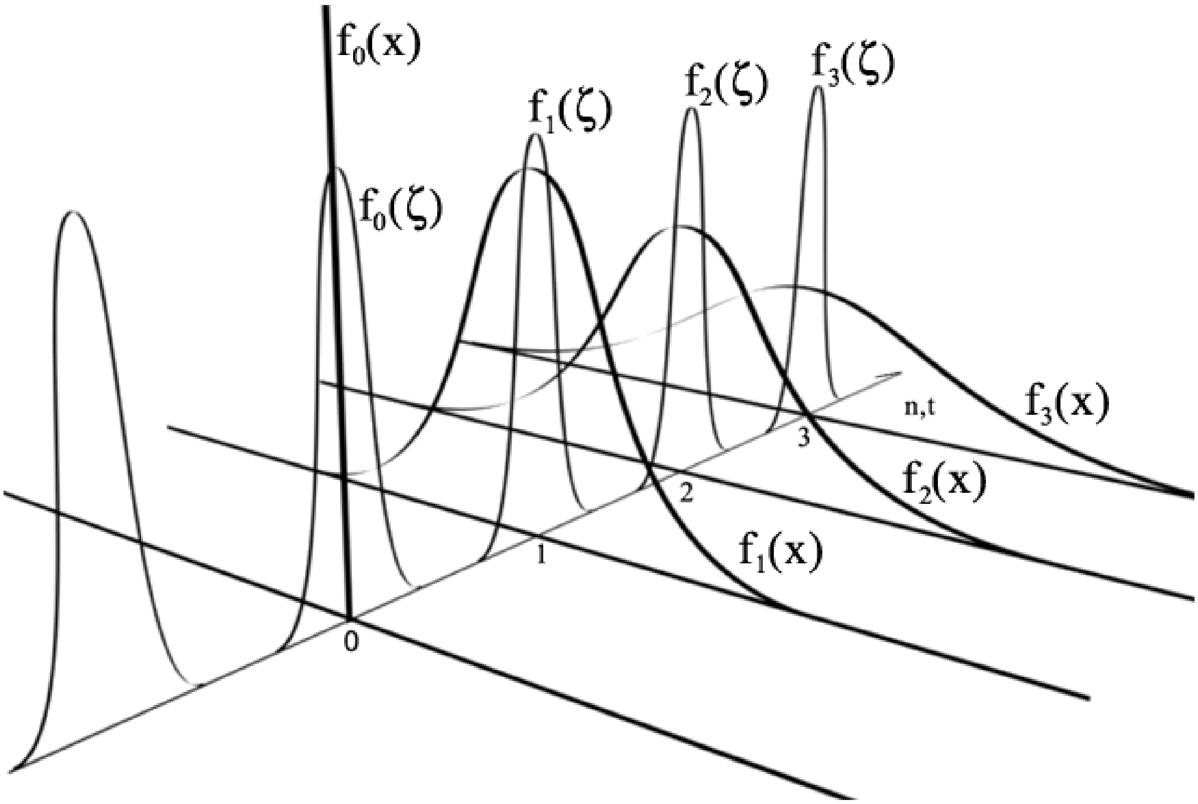

Figure 2. Schematic representation of distributions for ζ_n and x_n

Figure 2 gives a schematic representation of density distribution function $f_n(\zeta)$ and $f_n(x)$ at different time t_n. The density distribution function $f_n(\zeta)$ (representing transition from time t_{n-1} to t_n) is independent of t_n. Density distribution function $f_n(x)$, representing variable x, flattens out with time t_n. It must be pointed out that if the price generation process has autocorrelated increments, the volatility of price will be different from the volatility of the price increments. The volatility of a price with uncorrelated increments is equal to $\sigma^2 t$ at time t if the volatility of the price increments is equal to σ^2. However this relationship no longer holds if the returns are autocorrelated. The volatility measure $\Psi^2(t,\rho)$ defined earlier by

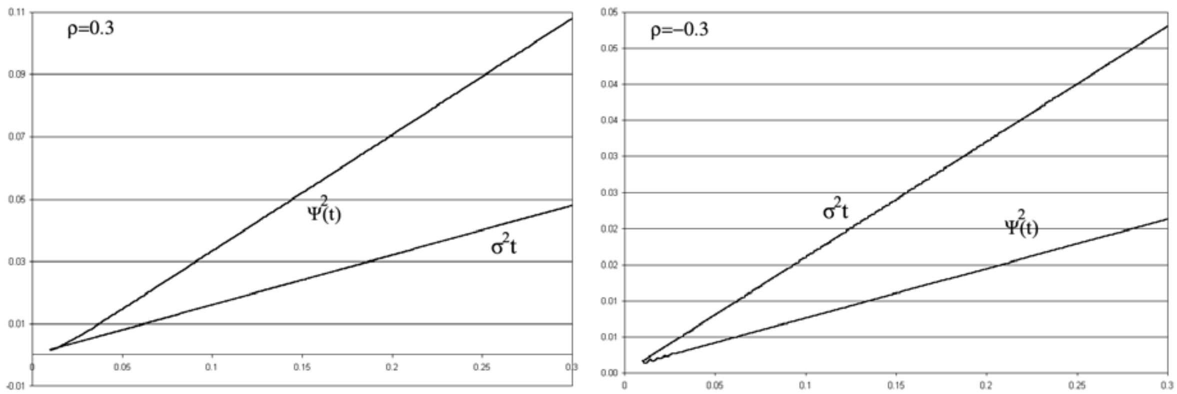

Figure 3. σ^2_{eff} and σ^2_{corr} as functions of time t
This figure presents different measures of volatility: variance of returns $\sigma^2 t$ and asset price variance $\Psi^2(t)$. Both graphs are obtained for $\sigma^2 = 0.4$

equation (8) measures the volatility of the price when σ^2 is the volatility of the autocorrelated increments. Figure 3 shows $\Psi^2(n,\rho)$, the volatility of variable x_n as a function of time t_n. The volatility $\sigma^2 \cdot t$ of a price with non-correlated increments is also presented. The assymptotic limit of $\Psi^2(n,\rho)$ is equal to $\sigma^2_{eff}(\rho)(T - sign(\rho)\cdot\tau_{corr})$. At time $t = 0$, $\Psi^2(0,\rho) \equiv 0$ and later it rapidly converges to its asymptotic value $\sigma^2_{eff}(\rho)(T - sign(\rho)\cdot\tau_{corr})$ for $t >> \tau_{corr}$. The values of $\Psi^2(n,\rho)$ and its assymptotic limit are almost identical for time $t >> \tau_{corr}$ and are represented by the same curve on Figure 3.

II. Option Pricing Model: Differential Approach

Let us proceed with derivation of option pricing formula when asset price is governed by the process described in the previous section. Assume that asset prices are log-normally distributed, i.e. $ln(S) = x$, where x's are normally distributed. The derivation of the option pricing model will be made using Ito stochastic calculus. To make a transformation from x to S in Ito calculus we need to use chain differentiation rule. In this particular case, Ito lemma cannot be employed because it is derived for the stochastic process following a Brownian motion. An extension of Ito calculus has to be derived for an autocorrelated sequence. The exact derivation is beyond the scope of this paper and can be found in Mezrin (2003). In that paper, for a stochastic process with autocorrelation coefficient ρ, chain differentiation rule for $(\Delta S)^2$ is found to be

$$(\Delta S)^2 = \sigma_{eff}^2 \left[1 - sign(\rho) \cdot \rho^n \frac{1-\rho}{ln|1/\rho|} \right] \Delta t. \tag{12}$$

For a continuous process with positive autocorrelation equation (12) can be written as

$$d^2 S = \sigma_{eff}^2 \left[1 - exp(-\frac{t}{\tau_{corr}}) \right] \cdot dt. \tag{13}$$

For a special case of a Brownian process ($\rho = 0$), equation (13) transforms to the well known result $(\Delta S)^2 = \sigma^2 \Delta t$ given by Ito lemma.

To obtain an option pricing formula applicable in the environment with interest rates, we add an interest rate term into equation (1). This new stochastic equation for x is given by

$$\Delta x_n = r\Delta t + \sqrt{\Delta t} \cdot \zeta_n(\sigma^2, \rho). \tag{14}$$

It must be pointed out that introduction of interest rate in the form $r\Delta t$ creates a positive autocorrelation in the process x. It can be shown that for small values of r, this additional term for autocorrelation is equal to

$$\frac{r}{\sigma^2} \cdot r\Delta t. \tag{15}$$

This term is relatively insignificant because it is proportional to the square of r and Δt, both of which are quite small. As usual, it will be neglected in the calculations to follow. However, the calculations can be easily extended to take it into consideration.

An option pricing formula can be obtained in two different ways. First one is by constructing a portfolio of options and stocks. After taking into account hedging, a differential equation can be derived as was done by Black Scholes (1973). The solution of this differential equation for certain boundary conditions will yield an option pricing formula. An alternative method involves using a density distribution function at time T and chain differential rule. Both of these methods will lead to identical results.

Let us construct a portfolio of stocks and options, where the weight is chosen to eliminate the short-term variations in stock price and ensure constant returns on portfolio.

$$\Pi = -C + \frac{\partial C}{\partial S} S \tag{16}$$

The change in the portfolio value is given by $\Delta\Pi = -\Delta C + \frac{\partial C}{\partial S}\Delta S$. Substituting the values of ΔC and ΔS and simplifying yields an expression similar to the Black Scholes equation

$$\left[\frac{\partial C}{\partial t} - rC + \frac{\partial C}{\partial S} rS\right] dt + \frac{1}{2}\frac{\partial^2 C}{\partial S^2} d^2 S = 0 \tag{17}$$

To obtain the option value we need to solve equation (17) where stock price S is distributed lognormally and given by a geometric stochastic equation with autocorrelated incerements.

$$\frac{\Delta S_n}{S_n} = r\Delta t + \sqrt{\Delta t} \cdot \zeta_n(\sigma^2, \rho). \tag{18}$$

Solving equation (17) with conditions (18) and chain differentiation rule (13) by means of Fourier transformation yields the end result

$$C_\rho = S_0 N(b_1) - Ee^{-rT} N(b_2), \tag{19}$$

where

$$b_1 = \frac{ln\frac{S_0}{E} + rT + \frac{1}{2}\left[\sigma^2_{eff}(\rho)T + \sigma^2_{corr}(\rho)\Delta T\right]}{\sqrt{\sigma^2_{eff}(\rho)T + \sigma^2_{corr}(\rho)\Delta T}}, \tag{20}$$

$$b_2 = b_1 - \sqrt{\sigma^2_{eff}(\rho)T + \sigma^2_{corr}(\rho)\Delta T}. \tag{21}$$

This is an exact analytical solution of geometric autocorrelated random walk stochastic equation (18).

III. Option Pricing Model: Integral Approach

An alternative approach is to use integral representation of the problem. This method is also be used to create a Monte Carlo simulation of the process. An option pricing formula can be obtained by integrating the value weighted distribution function of S at the expiration time $t_n = T$. A price C_ρ for a call option would be given as

$$C_\rho = e^{-rT} \int_{-\infty}^{\infty} max(S-E,0) \cdot f_\rho(S) d(lnS), \qquad (22)$$

where E is the strike price and $f_\rho(S)$ is given by

$$f_\rho(S) = \frac{1}{\sqrt{2\pi(\sigma_{eff}^2 T + \sigma_{corr}^2 \tau_{corr})}} exp\left[-\frac{\left[lnS - r \cdot T + \frac{1}{2}(\sigma_{eff}^2 T + \sigma_{corr}^2 \tau_{corr}\right]^2}{2(\sigma_{eff}^2 T + \sigma_{corr}^2 \tau_{corr})}\right], \qquad (23)$$

where we take into account the chain differentiation rule (13). Substituting (23) into (22) and integrating, yields the option pricing formula identical to equation (19)

The presence of $\sigma_{corr}^2(\rho)\tau_{corr}$ term in (20) and (21) can be explained by the behavior of $\Psi^2(t_n,\rho)$ on the interval $0 \leq t_n \leq (2-3)\tau_{corr}$. Because of the integral nature of the process it retains memory of its deterministic initial value S_0 on the time interval $0 \leq t \leq (2-3)\tau_{corr}$ when it has not randomized completely.

If the initial stages of the process are ignored, a limit equation can be defined when $T >> \tau_{corr}$ as

$$\tilde{C}_\rho = S_0 N(\tilde{b_1}) - E e^{-rT} N(\tilde{b_2}), \qquad (24)$$

where

$$\tilde{b_1} = \frac{ln\frac{S_0}{E} + rT + \frac{1}{2}\left[\sigma^2_{eff}(T - sign(\rho)\cdot\tau_{corr})\right]}{\sqrt{\sigma^2_{eff}(\rho)(T - sign(\rho)\cdot\tau_{corr})}}, \tag{25}$$

$$\tilde{b_2} = \tilde{b_1} - \sqrt{\sigma^2_{eff}(\rho)(T - sign(\rho)\cdot\tau_{corr})},\ \ \sigma^2_{eff}(\rho) = \frac{1+\rho}{1-\rho}\sigma^2. \tag{26}$$

For large T it might be practical to use truncated version of the model given by equation (24) instead of full version given by equation (19).

IV. Properties of Option Pricing Formula.

The option pricing formula (19) differs from famous Black Scholes option pricing formula in many aspects. Even for $n \to \infty$, their result will differ because the value of volatility $\sigma^2_{eff}(\rho)(T - sign(\rho)\cdot\tau_{corr})$ is greater than σ^2 for positive correlation coefficient and is less for a mean-reverting process as can be seen from Figure 1. The values of Greeks will also change due to differences in option valuation formulas. While the expression for Δ resembles the one obtained for Black-Scholes formula, Hull (1999),

$$\Delta = \frac{\partial\Pi}{\partial S} = N(b_1) \tag{27}$$

where the expression for b_1 is given by (20). At the same time the expression for Θ has changed significantly, becoming

$$\Theta = \frac{\partial\Pi}{\partial T} = -\frac{S_0 N'(b_1)\sigma^2_{eff}}{2\sqrt{\sigma^2_{eff}T + \sigma^2_{corr}\Delta T}} - rEe^{-rT}N(b_2) \tag{28}$$

The empirical test provide evidence of superior performance. The test results presented in Table (I) confirm that the difference between market value of the option and corresponding Black Scholes valuation is significantly larger than the difference between market value of the option and our price when the price of the stock has autocorrelated returns. The magnitude of the improvement is statistically significant, being on average equal to 26.6 cent per contract. The error is reduced on average by 58.53%. Monte Carlo simulation that is described in Section (V) shows the magnitude of errors when there are no additional factors impacting the option price.

Table I
Empirical Tests of Option Pricing Model

The value of the difference is calculated by taking the difference of errors between Black Scholes Option pricing formula and the model developed in Section (II). The tests are significant at 5% singificance level as indicated by the t statistic. The companies used have to exhibit autocorrelation of returns in excess of 0.1 and have actively traded options with more than 1000 contracts outstanding.

N	ΔC	$\Delta C/C_{BS}$	$Std.Dev.$	T - Value	P Value
147	0.2662	0.5853	0.06711	3.967086	0.000129

The values obtained using analytical solution are shown using a bold line in Figure 4 and Figure 5 for positive and negative autocorrelation coefficient respectively. For comparison purposes, same figures show values obtained from Black Scholes option formula (neglecting the autocorrelation of return).

V. Monte Carlo Simulation.

In this section we want to develop a Monte Carlo procedure capable of simulating a price path of an asset with autocorrelated returns. An evolution of the asset price S defined by the stochastic equation (14), after taking into consideration the fact that $lnS = x$. Numerical simulations employing equation (14) are inconvenient. It is common for Monte Carlo simulations to use an integral representation of (14) for S. In order to obtain it the chain differentiation

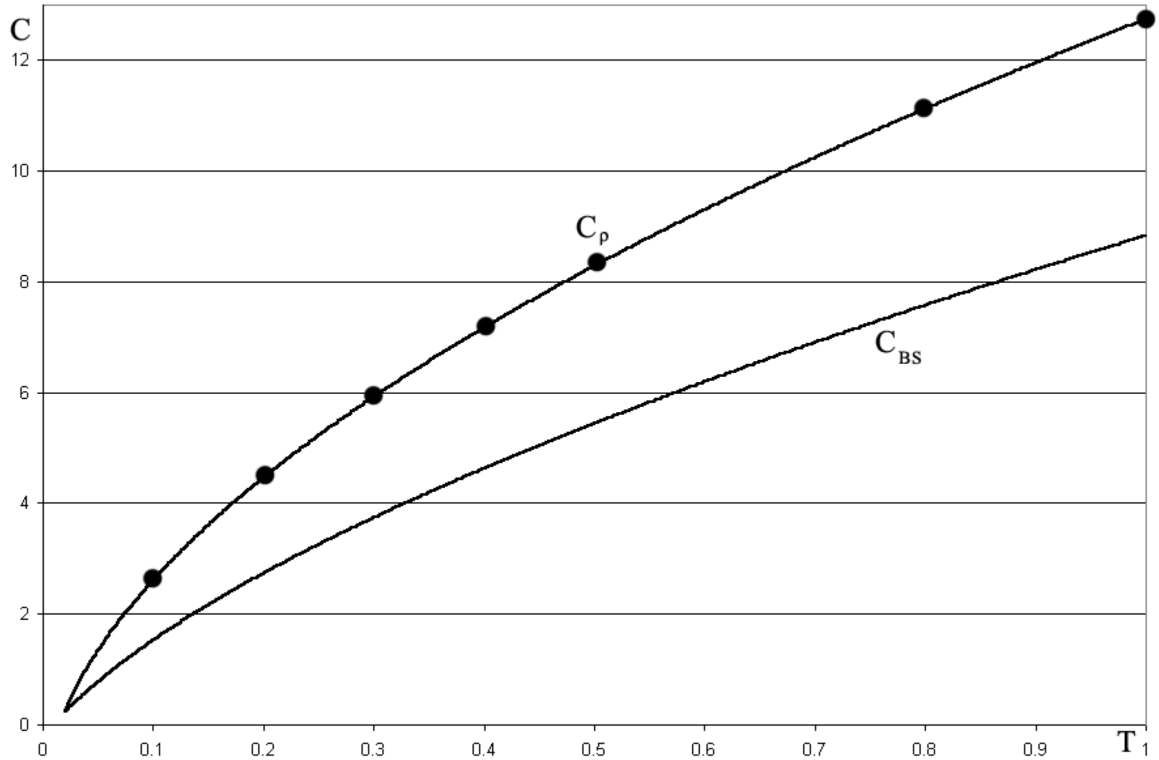

Figure 4. Option prices C_ρ, C_{BS} as functions of T for $\rho = 0.4$.
Results are presented for different models: Black Scholes model C_{BS} and exact analytical solution using equation (19) and C_ρ. Result of Monte Carlo simulation are shown as solid dots. Option properties: $S_0 = 50$, $E = 53$, $r = 0.1$, $\rho = 0.4$, $\sigma = 0.4$.

rule (13) has to be used. Therefore, an integral representation of the stochastic equation (14) for S can be written in form:

$$S_{n+1} = S_n \cdot exp\left[r \cdot \Delta t + \sqrt{\Delta t} \cdot \zeta_n(\sigma^2, \rho) - \frac{\sigma^2_{eff}(\rho)}{2}\left[\Delta t - sign(\rho) \cdot \rho^n \cdot (1-\rho) \cdot \tau_{corr}\right]\right]. \quad (29)$$

where $n = 0, 1, 2, ..., M$, $M = \frac{T}{\Delta t}$, and the initial value for (29) is S_0 and sequence of ζ_n's is given by

$$\zeta_n(\sigma^2, \rho) = \rho \cdot \zeta_{n-1}(\sigma^2, \rho) + \varepsilon_n((1-\rho)^2\sigma^2); \ n = -k, -k+1, ...0, 1, 2, ..., M; \ \ k \approx 20 - 30. \quad (30)$$

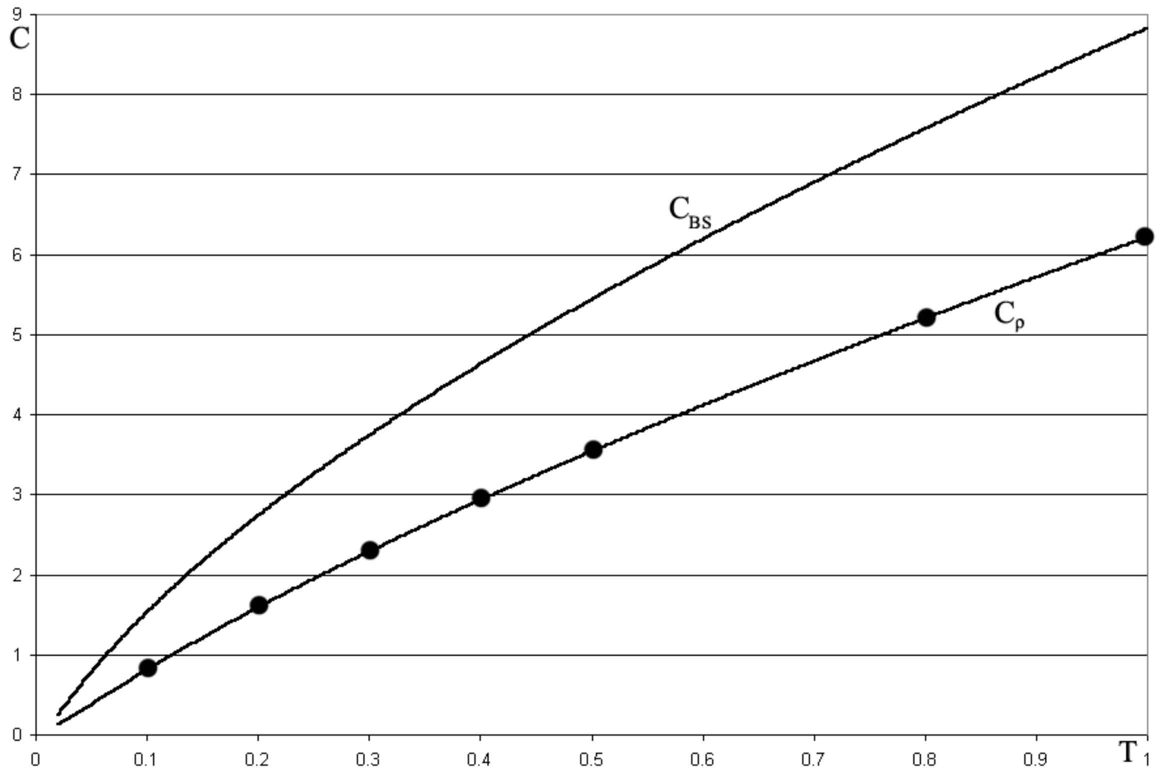

Figure 5. Option prices C_ρ, C_{BS} as functions of T for $\rho = -0.4$
Results are presented for different models: Black Scholes model C_{BS} and exact analytical solution using equation (19) C_ρ. Result of Monte Carlo simulation are shown as solid dots. Option properties: $S_0 = 50$, $E = 53$, $r = 0.1$, $\rho = -0.4$, $\sigma = 0.4$.

As can be seen from equation (30), ζ_n's are correlated with the correlation coefficient ρ. As was pointed out in the earlier section, in order for ζ to have volatility equal to σ^2, it must be preceded with k elements for some large value of k, i.e. ζ must start at ζ_{-k}. ($k \approx 20-30$)

The value of the call option is computed according to the formula

$$C_\rho = e^{-rT}\frac{1}{I}\sum_{i=1}^{I} max(S_i - E, 0), \tag{31}$$

where I is the number of simulated trajectories. To obtain reasonable results it is sufficient to take $I \approx 1-10$ million.

We would like to compare two different approaches derived earlier: an analytical solution (19) of stochastic equation (18) and simulation technique described above. Table II presents numerical values for analytical solution and Monte Carlo simulation. The Monte Carlo simulation used 100 million trajectories to simulate option price by utilizing equation (29) and (30) with both positive and negative autocorrelation. The results match to within the computational error (0.1%). This remarkable accuracy serves as a confirmation of both the analytical model and Monte Carlo simulation technique. Additionally, results of Monte Carlo simulation for different parameters are presented in Figure 4 and Figure 5 as solid dots. An even better accuracy can be obtained by increasing number of trajectories. Alternatively, a variational redundancy technique can be employed, e.g. Quasi-Monte Carlo method.

Table II
Option Prices obtained using numerical and analytical methods

Results are presented for different methods and measures of volatility: truncated option price model (equation (24)) $\tilde{C}_\rho$, Black Scholes model C_{BS}, full analytical option pricing model (equation (19)) C_ρ, Monte Carlo simulation $C_{MonteCarlo}$ for I=100 million trajectories. Option properties are: $S_0 = 50$, $E = 53$, $r = 0.1$, $\Delta t = 0.01$.

Positive Autocorrelation $\rho = 0.4$					Negative Autocorrelation $\rho = -0.4$				
T	C_{BS}	$\tilde{C}_\rho$	C_ρ	$C_{MonteCarlo}$	T	C_{BS}	$\tilde{C}_\rho$	C_ρ	$C_{MonteCarlo}$
0.1	1.53306	2.61549	2.61546	2.61623	0.1	1.53306	0.81166	0.81167	0.81158
0.2	2.74031	4.46749	4.46749	4.46811	0.2	2.74031	1.59145	1.59145	1.59139
0.3	3.74381	5.92466	5.92466	5.92534	0.3	3.74381	2.28990	2.28990	2.28982
0.4	4.63148	7.17404	7.17404	7.17523	0.4	4.71554	2.99633	2.99633	2.99621
0.5	5.44131	8.28826	8.28826	8.28942	0.5	5.44131	3.53968	3.53968	3.53957
0.8	7.57254	11.1258	11.1258	11.12595	0.8	7.57254	5.19940	5.19940	5.19933
1	8.82745	12.7418	12.7418	12.74311	1	8.82745	6.21538	6.21538	6.21521

VI. Conclusions and Further Research.

We have developed an analytical option pricing model applicable for the case when asset price returns are serially correlated. The resulting formula demonstrates a surprising relationship between asset price volatility, asset return volatility and asset return autocorrelation coefficient.

As a special case for no autocorrelation of returns, our model reduces to a well known Black Scholes options pricing formula. However, for the case of autocorrelated returns, our model is free of bias due to under or overestimation of asset price volatility.

While various extensions of the original Black Scholes option pricing formula have been created in the past, it was not our goal to create a model that would be superior to them. The purpose of this paper was to account the influence of autocorrelation in asset returns and determine the effects they have on the price of an option.

The model created here can be further extended to include factors such as stochastic volatility and interest rates as well as jump diffusion components. We believe that due to fact that this model controls for autocorrelation in asset returns, the results obtained should be superior when compared to similar models that do not control for autocorrelation.

References

[1] Andrews, Donald, 1993, Exactly Median-Unbiased Estimation of First Order Autoregressive/Unit Root Models, *Econometrica* 61, 139-165

[2] Amin, Kaushik, and Robert Jarrow, 1992, Option pricing on risky asset in a stochastic interest rate economy, *Mathematical Finance* 2, 217-237.

[3] Amin, Kaushik, and Victor Ng, 1993, Option valuation with systematic stochastic volatility, *Journal of Finance* 48, 881-910.

[4] Bakshi, Gurdip, and Zhiwu Chen, 1997, An alternative valuation model for contingent claims, *Journal of Financial Economics* 44, 123-165.

[5] Bates, David, 1991, The crash of 87: Was it expected? The evidence from options markets, *Journal of Finance* 46, 1009-1044.

[6] ———, 1996, Jumps and stochastic volatility: Exchange rate processing inplicit in Deutche mark options, *Review of Financial Studies* 9, 69-108.

[7] Conrad, Jenifer, and Gautam Kaul, 1988, Time-variation in expected returns, *Journal of Business* 61, 409-425.

[8] ———, 1989, Mean reversion in short-horizon expected returns, *Review of Financial Studies* 2, 225-240.

[9] Cox, John, and Stephen Ross, 1976, The valuation of options for alternative stochastic processes, *Journal of Financial Economics* 3, 145-166.

[10] Efron, B. 1979, Bootstrap method, another look at the jackknife, *The Annals of Statistics* 7, 1-26

[11] ———, B. 1982, The Jackknife, the Bootstrap and other resampling plans. (SIAM Philadelphia)

[12] Fama, Eugene, F. and Kenneth R. French, 1988, Permanent and temporary components of stock market prices, *Journal of Political Economy* 96, 246-273.

[13] Heston, Steven, 1993, A closed-form solution for options with stochastic volatility with application to bond and currency options, *Review of Financial Studies* 6, 327-343.

[14] Hull, John, and Alan White, 1987, The pricing of options with stochastic volatilities, *Journal of Finance* 42, 281-300.

[15] Hull, John, 1999, Options Futures and Other Derivatives, (Prentice-Hall, Inc)

[16] Lo, Andrew W. and Craig MacKinley, 1988, Stock market prices do not follow random walks: Evidence from simple specification test, *Review of Financial Studies* 1, 41-66.

[17] Madan, Dilip, and Eric Chang, 1996, The VG option pricing model, Working paper, university of Maryland and Georgia Institute of Technology.

[18] Melino, Angelo, and Stuart Turnbull, 1990, Pricing foreign currency options with stochastic volatility, *Journal of Econometrics* 45, 239-265.

[19] Merton, Robert, 1973, Theory of rational option pricing, *Bell Journal of Economics* 4, 141-183.

[20] ———, 1976, Option pricing when the underlying stock returns are discontinuous, *Journal of Financial Economics* 4, 125-144.

[21] Mezrin, Vadim, 2003, Extension of Ito Calculus for the case of autocorrelated random processes. Working paper, Rutgers university.

[22] Poterba, James, M. and Lawrence H. Summers, 1988, Mean reversion in stock prices: Evidence and implications, *Journal of Financial Economics* 22, 27-59.

[23] Schick A., 1998, An Adaptive Estimator of the Autocorrelation Coefficient in Regression Models with Autoregressive Errors, *Journal of Time Series Analysis* vol. 19,pp. 575-589

[24] Scott, Louis, 1987, Option pricing when the variance changes randomly: Theory, estimators and applications, *Journal of Financial and Quantitative Analysis* 22, 419-438.

Appendix A. Weak Form Solution for Autocorrelated Process

Consider the AR(1) proceses for ζ given by (3) with constant coefficient ρ and variance of stochastic component ε equal to $(1-\rho^2)\sigma^2$. Our next goal is to find the distribution of ζ. From (3) sequence of increments is given as

$$\begin{aligned}
\zeta_1 &= \varepsilon_1 \\
\zeta_2 &= \rho\varepsilon_1 + \varepsilon_2 \\
\zeta_3 = \rho(\varepsilon_2 + \rho\varepsilon_1) + \varepsilon_3 &= \varepsilon_3 + \rho\varepsilon_2 + \rho^2\varepsilon_1 \\
&\ldots \\
\zeta_n &= \varepsilon_n + \rho\varepsilon_{n-1} + \ldots + \rho^{n-1}\varepsilon_1
\end{aligned}$$

Thus, if the random shocks ε are distributed according to $\varepsilon \sim N(0, \sigma^2(1-\rho^2))$ then the distribution of ζ_n will be determined according to

$$\zeta_n \sim N(0, (1-\rho^2)(1+\rho^2+\rho^4+\ldots+\rho^{2n-2})\sigma^2) = N(0, (1-\rho^{2n})\cdot\sigma^2). \tag{A1}$$

Hence, ζ are distributed according to a normal distribution with volatility σ^2 for $n >> 1$.